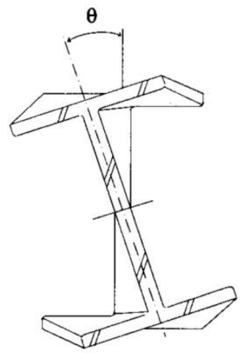
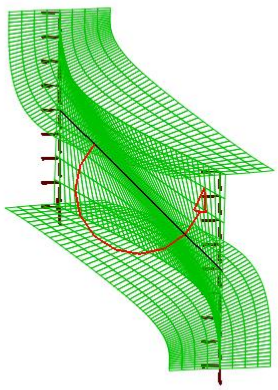
TORSION INCLUDING WARPING OF OPEN SECTIONS (I, C, Z, T AND L SHAPES)

Restrained warping for the torsion of thin-wall open sections is not included in most commonly used frame analysis programs. Almost all frame programs in practice use St-Venant torsion theory ignoring the effects of restrained warping.

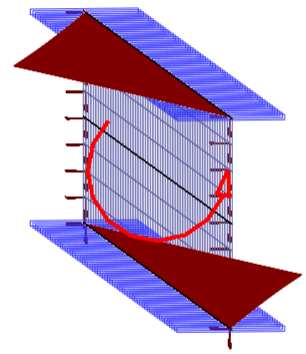


It is important to note that the torsional stiffness of an open section is function of the **warping end conditions** as well as the **location** of the torsional load. Thus, the distribution of the forces in the structure having members resisting torsion may differ whether this option is enabled or disabled. A subdivided continuous member needs to be specified as a **physical member** to get the continuity effect of warping along the member.

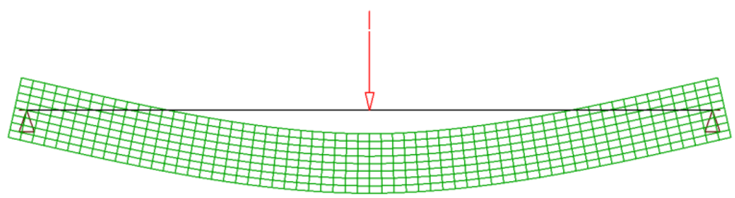
In addition to shear stresses, some members carry torque by axial stresses. This is called warping torsion. This happens when the cross-section wants to warp, i.e., displace axially, but is prevented from doing so during twisting of the beam. In other words, the section tends to resist torsion by out of plane bending of the flanges.

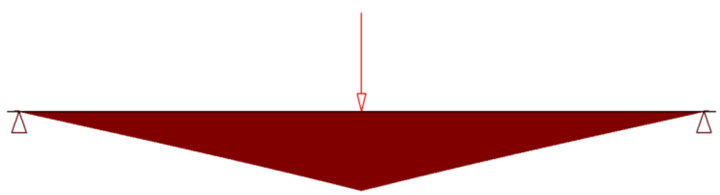


Deformation

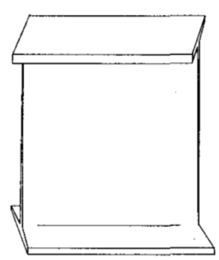


Bending in flanges





Equivalent view of the top flange



Warped section

Axial warping stresses

For solid and thin-wall closed sections (square, rectangular and circular tubes) these effects are often negligible. However, for thin-wall open sections restrained warping is often dominant. To include the effect of restrained warping we need to know the torsion constant J and the warping constant C_w which are geometric properties of the cross-section. For open shapes (I, C, Z, T, L, built-up …) the following equation may be used to evaluate the influence of the warping.

$$C = \sqrt{\frac{GJ}{EC_w}}$$

where

G	Shear modulus of elasticity of the material
J	Torsional constant for the cross section
E	Modulus of elasticity of the material
C_w	Warping constant for the cross-section

For axisymmetric cross-sections and thin-walled cross-sections with straight parts that intersect at one point such as X-shaped, T-shaped, and L-shaped cross-sections pure torsion will generally dominate over warping stresses. In these cases, torsional loads are carried by pure torsional shear stresses (St-Venant torsion) regardless of the boundary conditions.

LIMIT STATES CALCULATION INCLUDING TORSION

For open and closed cross sections, torsional stresses are evaluated from an elastic analysis. In the **Torsion - Warping** table, several columns describe in more details the pure torsion limit state.

Pure Torsion and Warping Limit States (9 columns hidden)

Table Commands View Selection

0 3	Member ID	Section Name	Comb ID	Fvy ksi	Fvx ksi	Mxp (pure) (kip-in)	τ_t Rounding ksi	τ_t Web ksi	τ_t Flange ksi	ULS Tors Rounding	ULS Tors Web	ULS Tors Flange	ULS Tors	Notes
	11	W36	1 - Factored	27.4028	27.4028	448.1442	8.9190	4.2013	8.4027	0.3255	0.1533	0.3409	0.3409	
	12	W36	1 - Factored	27.4028	27.4028	263.2479	5.2392	2.4679	4.9359	0.1912	0.0901	0.1960	0.1960	
	13	W36	1 - Factored	27.4028	27.4028	474.3845	9.4413	4.4473	8.8947	0.3445	0.1623	0.3651	0.3651	

The allowable stress in torsion is evaluated according to the selected design code. As torsional stresses are shear stresses, the torsional limit state is added directly to the shear limit state already calculated. The columns modified by this limit state are shown below in the **Limit State Summary** table.

Limit States Summary

Table Commands View Selection

0 3	Memb ID	Section Name	Comb ID	Maximum	Compres	Tension	Bending M+	Bending M-	Compres Bending	Tension Bending	Shear Vfx/Vrx	Shear Vfy/Vry	Torsion	Warping	Deflection	Notes
	11	W36	1 - Factored	0.8008	0.0000	0.0000	0.4567	0.0000	0.8008	0.8008	0.3409	0.5010	0.3409	0.3441		
	12	W36	1 - Factored	0.9221	0.0000	0.0000	0.5153	0.0000	0.9221	0.9221	0.1960	0.1235	0.1960	0.4068		
	13	W36	1 - Factored	0.9221	0.0000	0.0000	0.5153	0.0000	0.9221	0.9221	0.3651	0.5546	0.3651	0.4068		

LIMIT STATES CALCULATION INCLUDING WARPING DUE TO TORSION

The warping stresses are evaluated from an elastic analysis for open cross sections. In the **Torsion - Warping** table, several columns describe in more details the warping limit state.

Pure Torsion and Warping Limit States (11 columns hidden)

Table Commands View Selection

0 3	Member ID	Section Name	Comb ID	Mxw (warping) (kip-in)	τ_w Web ksi	τ_w Flange ksi	Fbw Comp ksi	Fbw Tens ksi	Bw (Bimoment) kip-in.2	σ_x (due to Bw) ksi	ULS Warping	Notes
	11	W36	1 - Factored	765.7520		0.9384	45.6713	45.6713	57714.3281	15.7174	0.3441	
	12	W36	1 - Factored	354.8228		0.4348	45.6713	45.6713	68222.7188	18.5792	0.4068	
	13	W36	1 - Factored	905.1772		1.1093	45.6713	45.6713	68222.7188	18.5792	0.4068	

Warping induces axial stresses as tension, compression and bending as well as shear stresses. These stresses thus have an influence on the compression-bending, tension-bending and shear limit states. The columns modified by this limit state are shown below in the **Limit State Summary** table.

Limit States Summary

Table Commands View Selection

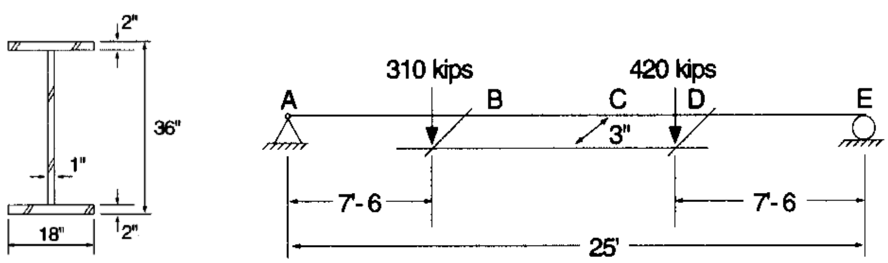
0 3	Memb ID	Section Name	Comb ID	Maximum	Compres	Tension	Bending M+	Bending M-	Compres Bending	Tension Bending	Shear Vfx/Vrx	Shear Vfy/Vry	Torsion	Warping	Deflection	Notes
	11	W36	1 - Factored	0.8008	0.0000	0.0000	0.4567	0.0000	0.8008	0.8008	0.3409	0.5010	0.3409	0.3441		
	12	W36	1 - Factored	0.9221	0.0000	0.0000	0.5153	0.0000	0.9221	0.9221	0.1960	0.1235	0.1960	0.4068		
	13	W36	1 - Factored	0.9221	0.0000	0.0000	0.5153	0.0000	0.9221	0.9221	0.3651	0.5546	0.3651	0.4068		

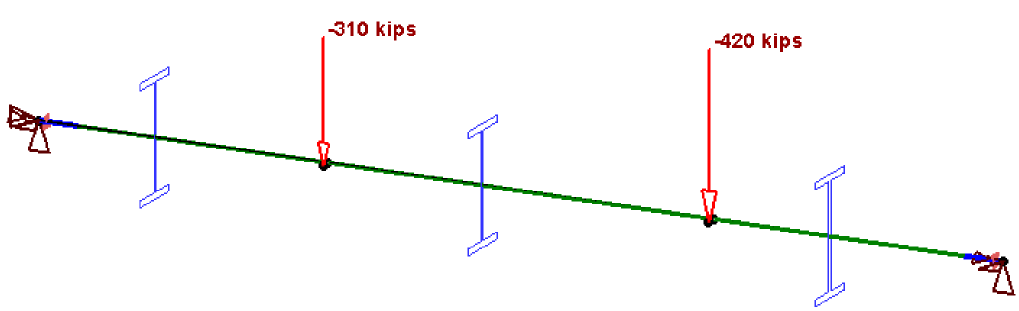
TORSION OF A SIMPLY SUPPORTED WELDED PLATE GIRDER

The following example is taken from example 5.4 of the reference *Torsional Analysis of Structural Steel Members, Steel Design Guide 9, AISC, Oct 2003*.

PROBLEM

A welded plate girder shown below with a span of 25 ft loaded with a 310 kips and a 420 kips factored loads (210 kips and 285 kips service loads). These concentrated loads are acting at a 3 in eccentricity with respect to the shear center. The end conditions are assumed to be flexurally and torsionally pinned.



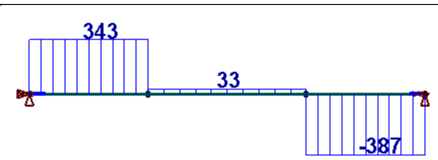


RESULTS COMPARISON

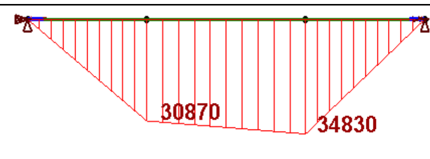
Location	Reference		SAFI	
Point D	σ_{bx} (ksi)	26.6	Sx (due to Mz)	26.61
D to E	τ_{bw}(ksi)	12.1	Txy (due to Fy)	12.15
D to E	τ_{bf}(ksi)	2.37	Not calculated	---
Point D	τ_t(ksi) for web	2.24	$\tau_t$web	2.20*
Point D	τ_t(ksi) for flange	4.47	$\tau_t$flange	4.40*
Point D	τ_t(ksi) in rounding	---	$\tau_t$rounding	5.03
Point E	τ_t(ksi) for web	4.37	$\tau_t$web	4.40*
Point E	τ_t(ksi) for flange	8.75	$\tau_t$flange	8.79*
Point E	τ_t(ksi) in rounding	---	$\tau_t$rounding	9.33
Point D	τ_w(ksi) due to warping	1.11	Txz (due to Mwx)	1.11
Point E	τ_w(ksi) due to warping	0.87	Txz (due to Mwx)	0.85
Midspan	σ_w(ksi) due to warping	18.4	Sx (due to Bw)	18.69
At 14.5 ft from A	Rotation under service load (degrees)	1.3	θ_x	1.34
At Center	Rotation under service load (degrees)	---	θ_x	1.37

* Displayed only in the steel verification results

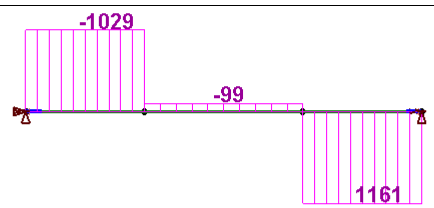
FORCES AND BENDING MOMENTS



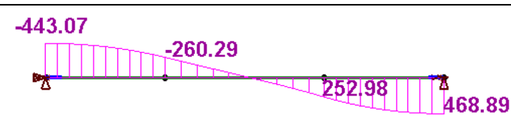
Shear Fy (kips)



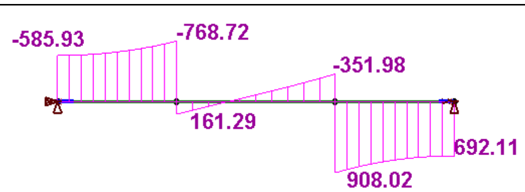
Bending Moment Mz (kip-in)



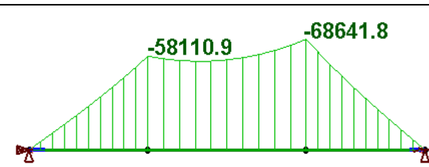
Mx (torsion) Total (kip-in)



Mxp (pure) torsion (kip-in)

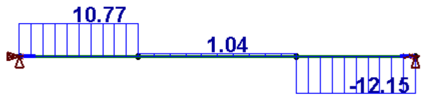


Mxw (warping) (kip-in)

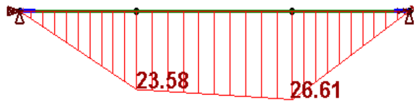


Bi-moment Bw (warping) (kip-in^2)

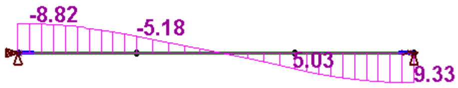
STRESSES



Txy (due to Fy) (ksi)

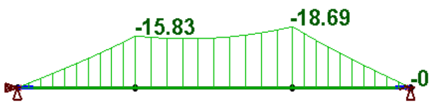


Sx (due to Mz) (ksi)



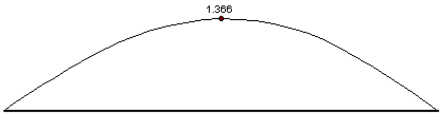
Tt (due to Mxp) (ksi)

Txz (due to Mxw) (ksi)



Sx (due to Bw) (ksi)

ROTATION



Rotation angle under service loads (degrees)

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